MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 (T2220)

EMT2066 – INDUSTRIAL MATHEMATICS

(All Sections / Groups)

5th JULY 2023

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **8 pages** (including cover page) with **4 questions** only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given..
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Several formulae are provided in the **Appendix** for your reference.

Question 1 (Numerical Methods)

(a) Apply Newton-Raphson method to estimate the root of the equation

$$2(x-0.95)+4x^3=0.$$

Start your iteration from $x_0 = 0.7$ and use $|x_n - x_{n-1}| < 10^{-2}$ as the termination criteria.

[10 marks]

(b) Consider the following system of linear equations:

$$\begin{aligned} 12x_1+3x_2-5x_3 &= 1 \\ -x_1-5x_2-3x_3 &= -28 \\ 3x_1+7x_2+13x_3 &= 76 \end{aligned}$$

By using Gauss-Seidel technique, estimate $[x_1, x_2, x_3]$ up to **two iterations** starting from an initial estimate $[x_1, x_2, x_3] = [1,0,1]$.

[15 marks]

Continued...

Question 2 (Stochastic Processes)

(a) A Markov chain with state space $S = \{1, 2, 3, 4\}$ is governed by the transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1/4 & 0 & 0 & 3/4 \end{bmatrix}.$$

(i) Draw the state transition diagram representing the chain.

[4 marks]

(ii) Decompose the state space into equivalent classes.

[2 marks]

(iii) Which states are transient and which are recurrent? Justify your answer.

[2 marks]

(iv) Determine the period of the recurrent states. Are these states periodic? Why or why not?

[3 marks]

(v) Evaluate $P_{42}^{(4)}$, the probability of moving from state 4 to state 2 in exactly 4 transitions.

[4 marks]

(vi) Suppose that the Markov chain have initial state probabilities $\mathbf{p}(0) = [1/2 \quad 1/2 \quad 0 \quad 0]$. In which state will the chain most likely be found after 2 transitions?

[3 marks]

(b) An elementary digital transmitter sends data as a stream of 0's and 1's. From the bit pattern, it is observed that bit 0 is followed by bit 1 with probability 0.75. Bit 1 is followed by another bit 1 with probability 0.5. Construct the transition probability matrix that models the output of the transmitter. Hence, determine the proportion of bit 1's in the data stream after an extended period of time.

[7 marks]

Continued...

Question 3 (Operations Research)

(a) Consider the following linear programming (LP) problem:

$$\begin{aligned} \text{Maximize } z = \; & x_1 - x_2 + 2x_3 \\ \text{subject to:} \quad & 3x_1 - x_2 + x_3 \leq 4 \quad \text{(Constraint 1)} \\ & x_2 + 2x_3 \leq 11 \quad \text{(Constraint 2)} \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

By introducing slack variables where appropriate, express the LP problem in **standard form**. Then solve the LP problem using the **simplex method**. Be sure to specify the **entering and leaving variables**, as well as the **elementary row operations** for each iteration. For the benefit of comparison, a partially-filled simplex tableau at the optimal stage is shown below:

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	3	0	0	4/3	1/3	9
?	0	?	?	?	?	?	5
?	0	?	?	?	?	?	1

[15 marks]

(b) Convert the above LP problem to its **dual**.

[5 marks]

(c) Using appropriate properties, deduce the optimal solution and optimal value for the dual problem in (b).

[3 marks]

(d) What would happen to the optimal value if:

(i) the right-hand-side (RHS) of Constraint 1 is increased by 1 unit?

(ii) the RHS of Constraint 2 is decreased by 1 unit?

[2 marks]

Continued...

Question 4 (Operations Research)

(a) Data packets originating from a computer terminal A are routed to another computer terminal I in four hops. As the network in **Figure Q4a** shows, a packet from terminal A may be relayed to one of two terminals (B or C) in the first hop, one of three terminals (D, E or F) in the second hop, and one of two terminals (G or H) in the third hop. Terminals G and H will then forward the data packet to the receiving terminal I.

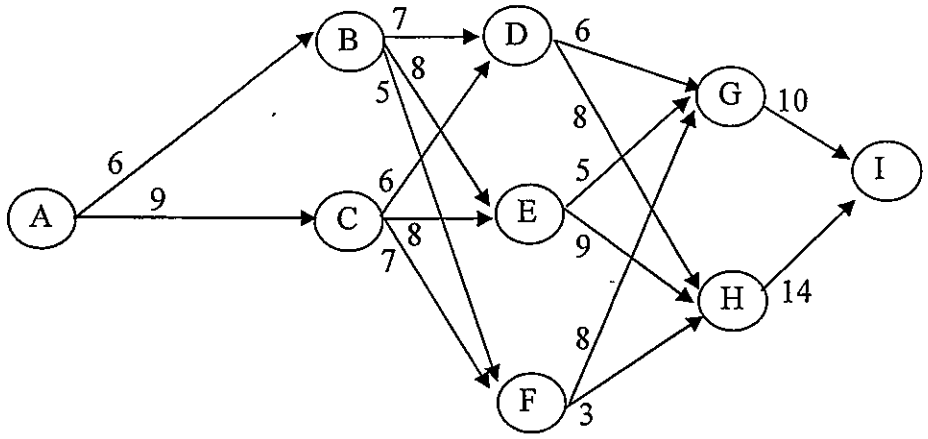

Figure Q4a

The number along each link represents the average delay (in milliseconds) experienced by a packet traversing that link. Using **dynamic programming**, determine a route from A to I that minimizes packet transmission time. What would this minimum transmission time be?

[16 marks]

(b) Using Kruskal's algorithm, demonstrate how the network in **Figure Q4b** may be reduced to a minimum spanning tree.

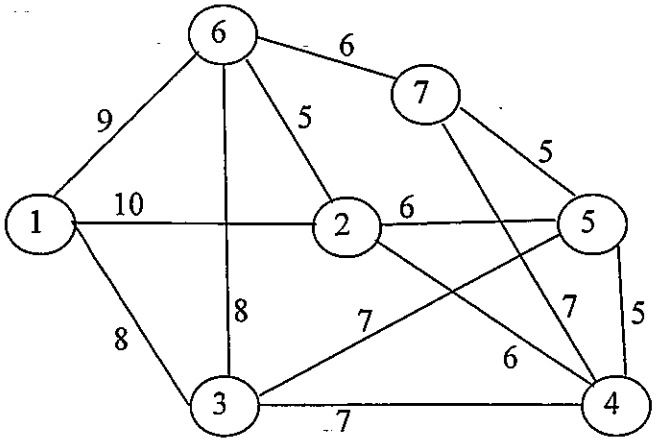

Figure 4b

[9 marks]

Continued...

APPENDIX: Selected Formulae for Numerical Methods

1. **Lagrange interpolating polynomial (LIP)**

$$P_n(x)=\sum_{i=0}^{n} L_i(x)f(x_i)$$

where $L_k(x)=\prod_{\substack{i=0\\ i\neq k}}^{n}\frac{(x-x_i)}{(x_k-x_i)}$.

2. **Newton's divided-difference interpolating polynomial (NDDIP)**

$$P_n(x)=f[x_0]+\sum_{k=1}^{n} f[x_0,x_1,...,x_k](x-x_0)\cdots(x-x_{k-1}).$$

3. **Newton's forward-difference interpolating polynomial**

$$P_n(x)=f[x_0]+\sum_{k=1}^{n}\binom{s}{k}\Delta^k f(x_0).$$

4. **Newton's backward-difference interpolating polynomial**

$$P_n(x)=f[x_n]+\sum_{k=1}^{n}(-1)^k\binom{-s}{k}\nabla^k f(x_n).$$

5. **Error in interpolating polynomial**

$$f(x)-P_n(x)=\frac{(x-x_0)(x-x_1)...(x-x_n)}{(n+1)!}f^{(n+1)}(c_x)$$

where $c_x \in [x_0, x_n]$.

6. **Forward difference formula for differentiation**

$$f'(x)\approx\frac{f(x+h)-f(x)}{h}.$$

7. **Backward difference formula for differentiation**

$$f'(x)\approx\frac{f(x)-f(x-h)}{h}.$$

8. **Central difference formula for differentiation**

$$f'(x)\approx\frac{f(x+h)-f(x-h)}{2h}$$

Continued...

9. **Trapezoidal rule**

$$\int_a^b f(x)dx \approx \frac{h}{2}\big(f(a)+f(b)\big).$$

where $h = b - a$.

10. **Composite Trapezoidal rule**

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right].$$

where $h = \dfrac{b-a}{n}$ and $x_j = a + jh,\ j = 0,1,\ldots,n$.

11. **Simpson's rule**

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right].$$

where $h = \dfrac{b-a}{2}$.

12. **Composite Simpson's rule**

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right].$$

where $h = \dfrac{b-a}{n}$ and $x_j = a + jh,\ j = 0,1,\ldots,n$.

13. **Gaussian Quadrature**

$$\int_a^b f(x)dx = \frac{(b-a)}{2}\int_{-1}^{1} f\left(x = \frac{(b+a)+t(b-a)}{2}\right)dt$$

$$\int_{-1}^{1} f(x)dx \approx w_1 f(x_1) + w_2 f(x_2) + \ldots + w_n f(x_n)$$

n	Nodes x_i	Weights, w_i
2	−0.57735 02692	1
	0.57735 02692	1
3	−0.77459 66692	0.5555 55556
	0	0.8888 88889
	0.77459 66692	0.5555 55556
4	−0.86113 63116	0.34785 48451
	−0.3399810436	0.65214 51549
	0.3399810436	0.65214 51549
	0.86113 63116	0.34785 48451

Continued...

14. **Newton-Raphson's method**

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})},\ n = 1,2,...$$

15. **Euler's method**

$$y_{i+1} = y_i + hf(x_i, y_i)$$

16. **Runge Kutta method of order two** (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

17. **Runge Kutta method of order four**

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right),$$
$$k_3 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

18. **Jacobi Method**

$$x_i^{(k)} = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{n} \left(-a_{ij} x_j^{(k-1)}\right) + b_i}{a_{ii}},\ \text{for } i = 1,2,...,n.$$

19. **Gauss-Siedel Method**

$$x_i^{(k)} = \frac{-\sum_{j=1}^{i-1} \left(a_{ij} x_j^{(k)}\right) - \sum_{j=i+1}^{n} \left(a_{ij} x_j^{(k-1)}\right) + b_i}{a_{ii}},\ \text{for } i = 1,2,...,n.$$

End of Paper.